Application of Derivatives

Exercise 6.2

Question 1:

Show that the function given by $f(x) = 3x + 17$ is strictly increasing on **R**.

Answer

Let x_1 and x_2 be any two numbers in **R**.

Then, we have:

$$x_1 < x_2 \Rightarrow 3x_1 < 3x_2 \Rightarrow 3x_1 + 17 < 3x_2 + 17 \Rightarrow f(x_1) < f(x_2)$$

Hence, f is strictly increasing on **R.**

Alternate method:

$f'(x) = 3 > 0$, in every interval of **R**.

Thus, the function is strictly increasing on **R**.

Question 2:

Show that the function given by $f(x) = e^{2x}$ is strictly increasing on **R**.

Answer

Let x_1 and x_2 be any two numbers in **R**.

Then, we have:

$$x_1 < x_2 \Rightarrow 2x_1 < 2x_2 \Rightarrow e^{2x_1} < e^{2x_2} \Rightarrow f(x_1) < f(x_2)$$

Hence, f is strictly increasing on **R**.

Question 3:

Show that the function given by $f(x) = \sin x$ is

(a) strictly increasing in $\left(0, \frac{\pi}{2}\right)$ (b) strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(c) neither increasing nor decreasing in $(0, \pi)$

Answer

The given function is $f(x) = \sin x$.

$$\therefore f'(x) = \cos x$$

(a) Since for each $x \in \left(0, \frac{\pi}{2}\right), \cos x > 0$, we have $f'(x) > 0$.

Hence, *f* is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

(b) Since for each $x \in \left(\frac{\pi}{2}, \pi\right), \cos x < 0$, we have $f'(x) < 0$.

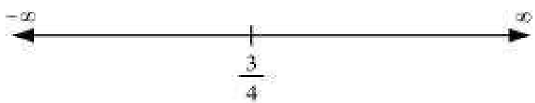
Hence, *f* is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.
(c) From the results obtained in (a) and (b), it is clear that *f* is neither increasing nor decreasing in (0, π).

Question 4:
Find the intervals in which the function *f* given by $f(x) = 2x^2 - 3x$ is
(a) strictly increasing (b) strictly decreasing
Answer
The given function is $f(x) = 2x^2 - 3x$.

$f'(x) = 4x - 3$

$\therefore f'(x) = 0 \Rightarrow x = \frac{3}{4}$

Now, the point $\frac{3}{4}$ divides the real line into two disjoint intervals i.e., $\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$.



In interval $\left(-\infty, \frac{3}{4}\right), f'(x) = 4x - 3 < 0.$

Hence, the given function (*f*) is strictly decreasing in interval $\left(-\infty, \frac{3}{4}\right)$.

In interval $\left(\frac{3}{4}, \infty\right), f'(x) = 4x - 3 > 0.$

Hence, the given function (*f*) is strictly increasing in interval $\left(\frac{3}{4},\infty\right)$.

Question 5:

Find the intervals in which the function *f* given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is

(a) strictly increasing (b) strictly decreasing

Answer

The given function is $f(x) = 2x^3 - 3x^2 - 36x + 7$.

$$f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x+2)(x-3)$$

$\therefore f'(x) = 0 \Rightarrow x = -2, 3$

The points $x = -2$ and $x = 3$ divide the real line into three disjoint intervals i.e.,

$(-\infty,-2),(-2,3)$, and $(3,\infty)$.

In intervals $(-\infty,-2)$ and $(3,\infty)$, $f'(x)$ is positive while in interval

$(-2, 3)$, $f'(x)$ is negative.

Hence, the given function (*f*) is strictly increasing in intervals

$(-\infty,-2)$ and $(3,\infty)$, while function (*f*) is strictly decreasing in interval $(-2, 3)$.

Question 6:

Find the intervals in which the following functions are strictly increasing or decreasing:

(a) $x^2 + 2x - 5$ (b) $10 - 6x - 2x^2$

(c) $-2x^3 - 9x^2 - 12x + 1$ (d) $6 - 9x - x^2$

(e) $(x + 1)^3 (x - 3)^3$

Answer

(a) We have,

$f(x)=x^2+2x-5$

$\therefore f'(x)=2x+2$

Now,

$f'(x)=0 \Rightarrow x = -1$

Point $x = -1$ divides the real line into two disjoint intervals i.e., $(-\infty,-1)$ and $(-1,\infty)$.

In interval $(-\infty,-1)$, $f'(x)=2x+2<0.$

$\therefore f$ is strictly decreasing in interval $(-\infty,-1)$.

Thus, f is strictly decreasing for $x < -1$.

In interval $(-1,\infty)$, $f'(x)=2x+2>0.$

$\therefore f$ is strictly increasing in interval $(-1,\infty)$.

Thus, f is strictly increasing for $x > -1$.

(b) We have,

$f(x) = 10 - 6x - 2x^2$

$\therefore f'(x)=-6-4x$

Now,

$f'(x)=0 \Rightarrow x=-\frac{3}{2}$

The point $x=-\frac{3}{2}$ divides the real line into two disjoint intervals

i.e., $\left(-\infty,-\frac{3}{2}\right)$ and $\left(-\frac{3}{2},\infty\right)$.

In interval $\left(-\infty,-\frac{3}{2}\right)$ i.e., when $x<-\frac{3}{2}$, $f'(x)=-6-4x<0.$

$\therefore$ *f* is strictly increasing for $x<-\frac{3}{2}$.

In interval $\left(-\frac{3}{2},\infty\right)$ i.e., when $x>-\frac{3}{2}$, $f'(x)=-6-4x<0.$

$\therefore$ *f* is strictly decreasing for $x>-\frac{3}{2}$.

(c) We have,

$f(x) = -2x^3 - 9x^2 - 12x + 1$

$\therefore f'(x)=-6x^2-18x-12=-6\left(x^2+3x+2\right)=-6(x+1)(x+2)$

Now,

$f'(x)=0 \Rightarrow x=-1 \text{ and } x=-2$

Points $x = -1$ and $x = -2$ divide the real line into three disjoint intervals

i.e., $(-\infty,-2),(-2,-1)$, and $(-1,\infty)$.

In intervals $(-\infty,-2)$ and $(-1,\infty)$ i.e., when $x < -2$ and $x > -1$,

$f'(x)=-6(x+1)(x+2)<0$.

∴ f is strictly decreasing for $x < -2$ and $x > -1$.

Now, in interval (−2, −1) i.e., when $-2 < x < -1$, $f'(x) = -6(x+1)(x+2) > 0$.

∴ f is strictly increasing for $-2 < x < -1$.

(d) We have,

$f(x) = 6 - 9x - x^2$

∴ $f'(x) = -9 - 2x$

Now, f'

$(x) = 0$ gives $x = -\frac{9}{2}$

The point $x = -\frac{9}{2}$ divides the real line into two disjoint intervals i.e.,

$\left(-\infty, -\frac{9}{2}\right)$ and $\left(-\frac{9}{2}, \infty\right)$.

In interval $\left(-\infty, -\frac{9}{2}\right)$ i.e., for $x < -\frac{9}{2}$, $f'(x) = -9 - 2x > 0$.

∴ f is strictly increasing for $x < -\frac{9}{2}$.

In interval $\left(-\frac{9}{2}, \infty\right)$ i.e., for $x > -\frac{9}{2}$, $f'(x) = -9 - 2x < 0$.

∴ f is strictly decreasing for $x > -\frac{9}{2}$.

(e) We have,

$f(x) = (x + 1)^3 (x - 3)^3$

$$f'(x) = 3(x+1)^2 (x-3)^3 + 3(x-3)^2 (x+1)^3$$
$$= 3(x+1)^2 (x-3)^2 [x-3+x+1]$$
$$= 3(x+1)^2 (x-3)^2 (2x-2)$$
$$= 6(x+1)^2 (x-3)^2 (x-1)$$

Now,

$f'(x) = 0 \Rightarrow x = -1,\ 3,\ 1$

The points $x = -1$, $x = 1$, and $x = 3$ divide the real line into four disjoint intervals

i.e., $(-\infty, -1)$, (−1, 1), (1, 3), and $(3, \infty)$.

In intervals $(-\infty, -1)$ and (−1, 1), $f'(x) = 6(x+1)^2 (x-3)^2 (x-1) < 0$.

∴ f is strictly decreasing in intervals $(-\infty, -1)$ and (−1, 1).

In intervals (1, 3) and $(3, \infty)$, $f'(x) = 6(x+1)^2 (x-3)^2 (x-1) > 0$.

∴ f is strictly increasing in intervals (1, 3) and $(3, \infty)$.

Question 7:

Show that $y = \log(1+x) - \frac{2x}{2+x}, x > -1$, is an increasing function of *x* throughout its domain.

Answer

We have,

$$y = \log(1+x) - \frac{2x}{2+x}$$

$$\therefore \frac{dy}{dx} = \frac{1}{1+x} - \frac{(2+x)(2) - 2x(1)}{(2+x)^2} = \frac{1}{1+x} - \frac{4}{(2+x)^2} = \frac{x^2}{(2+x)^2}$$

$$\text{Now, } \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{x^2}{(2+x)^2} = 0$$

$$\Rightarrow x^2 = 0 \qquad \left[(2+x) \neq 0 \text{ as } x > -1\right]$$

$$\Rightarrow x = 0$$

Since $x > -1$, point $x = 0$ divides the domain $(-1, \infty)$ in two disjoint intervals i.e., $-1 < x < 0$ and $x > 0$.

When $-1 < x < 0$, we have:

$$x < 0 \Rightarrow x^2 > 0$$

$$x > -1 \Rightarrow (2+x) > 0 \Rightarrow (2+x)^2 > 0$$

$$\therefore y' = \frac{x^2}{(2+x)^2} > 0$$

Also, when $x > 0$:

$$x > 0 \Rightarrow x^2 > 0, \ (2+x)^2 > 0$$

$$\therefore y' = \frac{x^2}{(2+x)^2} > 0$$

Hence, function *f* is increasing throughout this domain.

Question 8:

Find the values of x for which $y=\left[x(x-2)\right]^2$ is an increasing function.

Answer

We have,

$$y=\left[x(x-2)\right]^2=\left[x^2-2x\right]^2$$

$$\therefore \frac{dy}{dx}=y'=2\left(x^2-2x\right)(2x-2)=4x(x-2)(x-1)$$

$$\therefore \frac{dy}{dx}=0 \Rightarrow x=0, x=2, x=1.$$

The points x = 0, x = 1, and x = 2 divide the real line into four disjoint intervals i.e.,

$(-\infty,0)$, $(0,1)$ $(1,2)$, and $(2,\infty)$.

In intervals $(-\infty,0)$ and $(1,2)$, $\frac{dy}{dx}<0$.

∴ y is strictly decreasing in intervals $(-\infty,0)$ and $(1,2)$.

However, in intervals (0, 1) and (2, ∞), $\frac{dy}{dx}>0.$

∴ y is strictly increasing in intervals (0, 1) and (2, ∞).

∴ y is strictly increasing for $0 < x < 1$ and $x > 2$.

Question 9:

Prove that $y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$ is an increasing function of θ in $\left[0,\frac{\pi}{2}\right]$.

Answer

We have,

$$y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$$

$$\therefore \frac{dy}{dx}=\frac{(2+\cos\theta)(4\cos\theta)-4\sin\theta(-\sin\theta)}{(2+\cos\theta)^2}-1$$

$$=\frac{8\cos\theta+4\cos^2\theta+4\sin^2\theta}{(2+\cos\theta)^2}-1$$

$$=\frac{8\cos\theta+4}{(2+\cos\theta)^2}-1$$

$$\text{Now, }\frac{dy}{dx}=0.$$

$$\Rightarrow \frac{8\cos\theta+4}{(2+\cos\theta)^2}=1$$

$$\Rightarrow 8\cos\theta+4=4+\cos^2\theta+4\cos\theta$$

$$\Rightarrow \cos^2\theta-4\cos\theta=0$$

$$\Rightarrow \cos\theta(\cos\theta-4)=0$$

$$\Rightarrow \cos\theta=0 \text{ or } \cos\theta=4$$

Since cos $\theta \neq 4$, cos $\theta = 0$.

$$\cos\theta=0\Rightarrow\theta=\frac{\pi}{2}$$

Now,

$$\frac{dy}{dx}=\frac{8\cos\theta+4-\left(4+\cos^2\theta+4\cos\theta\right)}{(2+\cos\theta)^2}=\frac{4\cos\theta-\cos^2\theta}{(2+\cos\theta)^2}=\frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$$

In interval $\left(0,\ \frac{\pi}{2}\right)$, we have cos θ > 0. Also, 4 > cos $\theta \Rightarrow 4 - \cos\theta > 0$.

$$\therefore \cos\theta(4-\cos\theta)>0 \text{ and also } (2+\cos\theta)^2>0$$
$$\Rightarrow \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}>0$$
$$\Rightarrow \frac{dy}{dx}>0$$

Therefore, y is strictly increasing in interval $\left(0,\ \frac{\pi}{2}\right)$.

Also, the given function is continuous at $x=0$ and $x=\frac{\pi}{2}$.

Hence, y is increasing in interval $\left[0, \frac{\pi}{2}\right]$.

Question 10:

Prove that the logarithmic function is strictly increasing on (0, ∞).

Answer

The given function is $f(x)=\log x$.

$$\therefore f'(x)=\frac{1}{x}$$

It is clear that for $x > 0$, $f'(x)=\frac{1}{x}>0$.

Hence, $f(x) = \log x$ is strictly increasing in interval (0, ∞).

Question 11:

Prove that the function f given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on (−1, 1).

Answer

The given function is $f(x) = x^2 - x + 1$.

$\therefore f'(x) = 2x - 1$

Now, $f'(x) = 0 \Rightarrow x = \frac{1}{2}$.

The point $\frac{1}{2}$ divides the interval (−1, 1) into two disjoint intervals

i.e., $\left(-1, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, 1\right)$.

Now, in interval $\left(-1, \frac{1}{2}\right)$, $f'(x) = 2x - 1 < 0$.

Therefore, f is strictly decreasing in interval $\left(-1, \frac{1}{2}\right)$.

However, in interval $\left(\frac{1}{2}, 1\right)$, $f'(x) = 2x - 1 > 0$.

Therefore, f is strictly increasing in interval $\left(\frac{1}{2}, 1\right)$.
Hence, f is neither strictly increasing nor decreasing in interval (−1, 1).

Question 12:

Which of the following functions are strictly decreasing on $\left(0, \frac{\pi}{2}\right)$?
(A) cos x (B) cos $2x$ (C) cos $3x$ (D) tan x
Answer

(A) Let $f_1(x) = \cos x$.

$\therefore f_1'(x) = -\sin x$

In interval $\left(0, \frac{\pi}{2}\right)$, $f_1'(x) = -\sin x < 0$.

$\therefore f_1(x) = \cos x$ is strictly decreasing in interval $\left(0, \frac{\pi}{2}\right)$.

(B) Let $f_2(x) = \cos 2x.$

$\therefore f_2'(x) = -2\sin 2x$

Now, $0 < x < \frac{\pi}{2} \Rightarrow 0 < 2x < \pi \Rightarrow \sin 2x > 0 \Rightarrow -2\sin 2x < 0$

$\therefore f_2'(x) = -2\sin 2x < 0 \ on \left(0, \frac{\pi}{2}\right)$

$\therefore f_2(x) = \cos 2x$ is strictly decreasing in interval $\left(0, \frac{\pi}{2}\right)$.

(C) Let $f_3(x) = \cos 3x.$

$\therefore f_3'(x) = -3\sin 3x$

Now, $f_3'(x) = 0.$

$\Rightarrow \sin 3x = 0 \Rightarrow 3x = \pi, as\ x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow x = \frac{\pi}{3}$

The point $x = \frac{\pi}{3}$ divides the interval $\left(0, \frac{\pi}{2}\right)$ into two disjoint intervals

i.e., 0 $\left(0, \frac{\pi}{3}\right)$ and $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

Now, in interval $\left(0, \frac{\pi}{3}\right), f_3(x) = -3\sin 3x < 0 \left[as\ 0 < x < \frac{\pi}{3} \Rightarrow 0 < 3x < \pi\right]$.

$\therefore f_3$ is strictly decreasing in interval $\left(0, \frac{\pi}{3}\right)$.

However, in interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right), f_3(x) = -3\sin 3x > 0 \left[as\ \frac{\pi}{3} < x < \frac{\pi}{2} \Rightarrow \pi < 3x < \frac{3\pi}{2}\right]$.

$\therefore f_3$ is strictly increasing in interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

Hence, f_3 is neither increasing nor decreasing in interval $\left(0, \frac{\pi}{2}\right)$.

(D) Let $f_4(x) = \tan x$.

$\therefore f_4'(x) = \sec^2 x$

In interval $\left(0, \frac{\pi}{2}\right), f_4'(x) = \sec^2 x > 0.$

$\therefore f_4$ is strictly increasing in interval $\left(0, \frac{\pi}{2}\right)$.

Therefore, functions cos x and cos $2x$ are strictly decreasing in $\left(0, \frac{\pi}{2}\right)$.

Hence, the correct answers are A and B.

Question 13:

On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$ strictly decreasing?

(A) $(0, 1)$

(B) $\left(\frac{\pi}{2}, \pi\right)$

(C) $\left(0, \frac{\pi}{2}\right)$

(D) None of these

Answer

We have,

$f(x) = x^{100} + \sin x - 1$

$\therefore f'(x) = 100x^{99} + \cos x$

In interval $(0, 1)$, $\cos x > 0$ and $100x^{99} > 0$.

$\therefore f'(x) > 0$.

Thus, function *f* is strictly increasing in interval (0, 1).

In interval $\left(\frac{\pi}{2}, \pi\right)$, $\cos x < 0$ and $100x^{99} > 0$. Also, $100x^{99} > \cos x$

$\therefore f'(x) > 0$ in $\left(\frac{\pi}{2}, \pi\right)$.

Thus, function *f* is strictly increasing in interval $\left(\frac{\pi}{2}, \pi\right)$.

In interval $\left(0, \frac{\pi}{2}\right)$, $\cos x > 0$ and $100x^{99} > 0$.

$\therefore 100x^{99} + \cos x > 0$

$\Rightarrow f'(x) > 0$ on $\left(0, \frac{\pi}{2}\right)$

$\therefore$ *f* is strictly increasing in interval $\left(0, \frac{\pi}{2}\right)$.

Hence, function *f* is strictly decreasing in none of the intervals.

The correct answer is D.

Question 14:

Find the least value of *a* such that the function *f* given $f(x) = x^2 + ax + 1$ is strictly increasing on (1, 2).

Answer

We have,

$f(x) = x^2 + ax + 1$

$\therefore f'(x) = 2x + a$

Now, function *f* will be increasing in (1, 2), if $f'(x) > 0$ in (1, 2).

$f'(x) > 0$

$\Rightarrow 2x + a > 0$

$\Rightarrow 2x > -a$

$\Rightarrow x > \frac{-a}{2}$

Therefore, we have to find the least value of *a* such that

$$x > \frac{-a}{2}, \text{ when } x \in (1, 2).$$
$$\Rightarrow x > \frac{-a}{2} \text{ (when } 1 < x < 2)$$

Thus, the least value of *a* for *f* to be increasing on (1, 2) is given by,

$\frac{-a}{2} = 1$

$\frac{-a}{2} = 1 \Rightarrow a = -2$

Hence, the required value of *a* is −2.

Question 15:

Let **I** be any interval disjoint from (−1, 1). Prove that the function *f* given by

$f(x) = x + \frac{1}{x}$ is strictly increasing on **I**.

Answer

We have,

$$f(x) = x + \frac{1}{x}$$

$$\therefore f'(x) = 1 - \frac{1}{x^2}$$

Now,

$$f'(x) = 0 \Rightarrow \frac{1}{x^2} = 1 \Rightarrow x = \pm 1$$

The points $x = 1$ and $x = -1$ divide the real line in three disjoint intervals i.e.,

$(-\infty, -1), (-1, 1)$, and $(1, \infty)$.

In interval (−1, 1), it is observed that:

$-1 < x < 1$

$\Rightarrow x^2 < 1$

$\Rightarrow 1 < \frac{1}{x^2}, \ x \neq 0$

$\Rightarrow 1 - \frac{1}{x^2} < 0, \ x \neq 0$

$\therefore f'(x) = 1 - \frac{1}{x^2} < 0$ on $(-1, 1) \sim \{0\}$.

∴ *f* is strictly decreasing on $(-1, 1) \sim \{0\}$.

In intervals $(-\infty, -1)$ and $(1, \infty)$, it is observed that:

$x < -1$ or $1 < x$

$\Rightarrow x^2 > 1$

$\Rightarrow 1 > \frac{1}{x^2}$

$\Rightarrow 1 - \frac{1}{x^2} > 0$

$\therefore f'(x) = 1 - \frac{1}{x^2} > 0$ on $(-\infty, -1)$ and $(1, \infty)$.

∴ *f* is strictly increasing on $(-\infty, 1)$ and $(1, \infty)$.

Hence, function *f* is strictly increasing in interval **I** disjoint from (−1, 1).

Hence, the given result is proved.

Question 16:

Prove that the function *f* given by *f*(*x*) = log sin *x* is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

Answer

We have,

$f(x) = \log \sin x$

$\therefore f'(x) = \frac{1}{\sin x} \cos x = \cot x$

In interval $\left(0, \frac{\pi}{2}\right)$, $f'(x) = \cot x > 0$.

∴ *f* is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

In interval $\left(\frac{\pi}{2}, \pi\right), f'(x) = \cot x < 0.$

∴*f* is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

Question 17:

Prove that the function *f* given by *f*(*x*) = log cos *x* is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

Answer

We have,

$f(x) = \log \cos x$

$\therefore f'(x) = \frac{1}{\cos x}(-\sin x) = -\tan x$

In interval $\left(0, \frac{\pi}{2}\right)$, $\tan x > 0 \Rightarrow -\tan x < 0.$

$\therefore f'(x) < 0$ on $\left(0, \frac{\pi}{2}\right)$

∴*f* is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

In interval $\left(\frac{\pi}{2}, \pi\right)$, $\tan x < 0 \Rightarrow -\tan x > 0.$

$\therefore f'(x) > 0$ on $\left(\frac{\pi}{2}, \pi\right)$

$\therefore f$ is strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

Question 18:

Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in **R**.

Answer

We have,

$$f(x) = x^3 - 3x^2 + 3x - 100$$

$$\begin{aligned} f'(x) &= 3x^2 - 6x + 3 \\ &= 3(x^2 - 2x + 1) \\ &= 3(x-1)^2 \end{aligned}$$

For any $x \in$ **R**, $(x - 1)^2 > 0$.

Thus, $f'(x)$ is always positive in **R**.

Hence, the given function (f) is increasing in **R**.

Question 19:

The interval in which $y = x^2 e^{-x}$ is increasing is

(A) $(-\infty, \infty)$ (B) $(-2, 0)$ (C) $(2, \infty)$ (D) $(0, 2)$

Answer

We have,

$y = x^2 e^{-x}$

$$\therefore \frac{dy}{dx} = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2-x)$$

$$\text{Now}, \frac{dy}{dx} = 0.$$

$$\Rightarrow \ x = 0 \text{ and } x = 2$$

The points x = 0 and x = 2 divide the real line into three disjoint intervals

i.e., $(-\infty,\ 0),\ (0,\ 2)$, and $(2,\ \infty)$.

In intervals $(-\infty,\ 0)$ and $(2,\ \infty)$, $f'(x) < 0$ as e^{-x} is always positive.

∴f is decreasing on $(-\infty,\ 0)$ and $(2,\ \infty)$.

In interval (0, 2), $f'(x) > 0$.

∴ f is strictly increasing on (0, 2).

Hence, f is strictly increasing in interval (0, 2).

The correct answer is D.