Mathematics

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(Chapter – 1)(Number Systems)

(Class – 9)

Exercise 1.3

Question 1:

Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$ **(ii)** $\frac{1}{11}$ **(iii)** $4\frac{1}{8}$

(iv) $\frac{3}{13}$ **(v)** $\frac{2}{11}$ **(vi)** $\frac{329}{400}$

Answer 1:

(i) $\frac{36}{100} = 0.3$, Terminating.

(ii) $\frac{1}{11} = 0.\overline{09}$, Recurring & Non-terminating.

(iii) $4\frac{1}{8} = 4.125$, Terminating.

(iv) $\frac{3}{13} = 0.\overline{230769}$, Recurring & Non-terminating.

(v) $\frac{2}{11} = 0.\overline{18}$, Recurring & Non-terminating.

(vi) $\frac{329}{400} = 0.8225$, Terminating.

Question 2:

You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$ are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

Answer 2:

Without actual long division, the decimal expansions of $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$ are as follows:

$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$

$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$

$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$

$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$

$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$

Question 3:

Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) $0.\bar{6}$ **(ii)** $0.4\bar{7}$ **(iii)** $0.\overline{001}$

Answer 3:

(i) $0.\bar{6}$

Let $x = 0.\bar{6}$

$\Rightarrow x = 0.6666 \ldots$ ... (i)

Multiplying equation (i) by 10 both sides

$10x = 6.6666 \ldots$

$\Rightarrow 10x = 6 + 0.6666 \ldots$

$\Rightarrow 10x = 6 + x$ [From equation (i)]

$\Rightarrow 10x - x = 6$

$\Rightarrow 9x = 6$

$\Rightarrow x = \frac{6}{9} = \frac{2}{3}$

(ii) $0.4\bar{7}$

Let $x = 0.4\bar{7}$

$\Rightarrow x = 0.47777 \ldots$ ... (i)

Multiplying equation (i) by 10 both sides

$\Rightarrow 10x = 4.7777 \ldots$ ... (ii)

Multiplying equation (ii) by 10 both sides

$100x = 47.7777 \ldots$

$\Rightarrow 100x = 43 + 4.7777 \ldots$

$\Rightarrow 100x = 43 + 10x$ [From equation (ii)]

$\Rightarrow 100x - 10x = 43$

$\Rightarrow 90x = 43$

$\Rightarrow x = \frac{43}{90}$

(iii) $0.\overline{001}$

Let $x = 0.\overline{001}$

$\Rightarrow x = 0.001001001 \ldots$ ... (i)

Multiplying equation (i) by 1000 both sides

$1000x = 1.001001001 \ldots$

$\Rightarrow 1000x = 1 + 0.001001001 \ldots$

$\Rightarrow 1000x = 1 + x$ [From equation (i)]

$\Rightarrow 1000x - x = 1$

$\Rightarrow 999x = 1$

$\Rightarrow x = \frac{1}{999}$

Question 4:

Express $0.99999\ldots$ in the form of $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Answer 4:

$0.99999\ldots$

Let $x = 0.99999\ldots$ ... (i)

Multiplying equation (i) by 10 both sides

$10x = 9.99999\ldots$

$\Rightarrow 10x = 9 + 0.99999\ldots\ldots$

$\Rightarrow 10x = 9 + x$ [From equation (i)]

$\Rightarrow 10x - x = 9$

$\Rightarrow 9x = 9$

$\Rightarrow x = \frac{9}{9} = 1$

The answer makes sense as $0.99999\ldots$ is very close to 1, that is why we can say that $0.99999 = 1$.

Question 5:

What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer?

Answer 5:

The maximum number of digits that can be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$ is 16 (less than 17).

By performing the actual division, we get

$$\frac{1}{17} = 0.\overline{0588235294117647}$$

So, the maximum number of digits that can be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$ is 16.

Question 6:

Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Answer 6:

$\frac{2}{5} = 0.4$, $\frac{1}{10} = 0.1$, $\frac{3}{2} = 1.5$, $\frac{7}{8} = 0.875$

The denominator of all the rational numbers are in the form of $2^m \times 5^n$, where m and n are integers.

Question 7:

Write three numbers whose decimal expansions are non-terminating non-recurring.

Answer 7:

Three non-terminating non-recurring decimals:

1) 0.414114111411114 ...
2) 2.01001000100001 ...
3) $\pi = 3.1416$ ...

Question 8:

Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Answer 8:

$$\frac{5}{7} = 0.\overline{714285} \text{ and } \frac{9}{11} = 0.\overline{81}$$

We know that there are infinite many irrational numbers between two rational numbers.
So the three irrational numbers are:

1) 0.72722722272222 ...
2) 0.73733733373333 ...
3) 0.74744744474444 ...

Question 9:

Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$ **(ii)** $\sqrt{225}$ **(iii)** 0.3796

(iv) 7.478478 ... **(v)** 1.101001000100001 ...

Answer 9:

(i) $\sqrt{23}$, Irrational number

(ii) $\sqrt{225} = 15$, Rational number

(iii) 0.3796, Rational number

(iv) 7.478478 ... $= 7.\overline{478}$, Rational number

(v) 1.101001000100001 ..., Irrational number