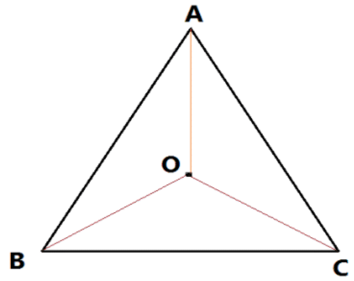
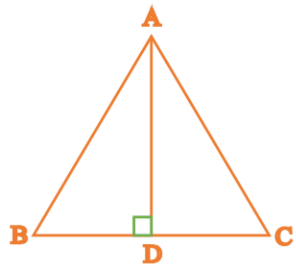
Exercise 7.2

Question 1:
In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that:
(i) OB = OC
(ii) AO bisects ∠A



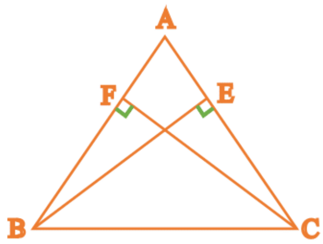
Answer 1:
(i) In ABC, AB = AC [∵ Given]
Hence, ∠ACB = ∠ABC [∵ Angles opposite to equal sides are equal]
⇒ $\frac{1}{2}$∠ACB = $\frac{1}{2}$∠ABC
⇒ ∠ACO = ∠ABO [∵ OB and OC bisect ∠B and ∠C respectively]
In ΔABO and ΔACO,
AB = AC [∵ Given]
∠ABO = ∠ACO [∵ Proved above]
AO = AO [∵ Common]
Hence, ΔABO ≅ ΔACO [∵ SAS Congruency Rule]
OB = OC [∵ CPCT]
(ii) ΔABO ≅ ΔACO [∵ Proved above]
∠BAO = ∠CAO [∵ CPCT]
Hence, OA bisects angle A.

Question 2:
In ΔABC, AD is the perpendicular bisector of BC (see Figure). Show that ΔABC is an isosceles triangle in which AB = AC.



Answer 2:
In ΔABD and ΔACD,
BD = DC [∵ AD bisects BC]
∠ADB = ∠ADC [∵ Each 90°]
AD = AD [∵ Common]
Hence, ΔABD ≅ ΔACD [∵ SAS Congruency Rule]
AB = AC [∵ CPCT]
Hence, ΔABC is an isosceles triangle.

Question 3:
ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Figure). Show that these altitudes are equal.



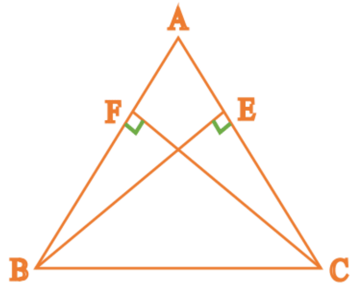
Answer 3:
In ΔABE and ΔACF,
∠AEB = ∠AFC [∵ Each 90°]
∠A = ∠A [∵ Common]
AB = AC [∵ Given]
Hence, ΔABE ≅ ΔACF [∵ AAS Congruency Rule]
BE = CF [∵ CPCT]

Question 4:

ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Figure). Show that

(i) $\Delta ABE \cong \Delta ACF$

(ii) $AB = AC$, i.e., ABC is an isosceles triangle.



Answer 4:

(i) In ΔABE and ΔACF,

$\angle AEB = \angle AFC$ [∵ Each 90°]

$\angle A = \angle A$ [∵ Common]

$BE = CF$ [∵ Given]

Hence, $\Delta ABE \cong \Delta ACF$ [∵ AAS Congruency Rule]

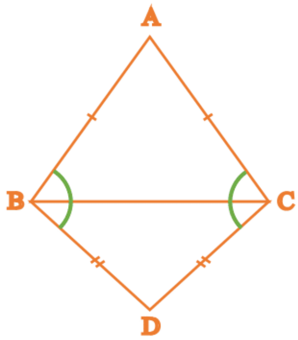
(ii) In $\Delta ABE \cong \Delta ACF$ [∵ Proved above]

$AB = AC$ [∵ CPCT]

Hence, ΔABC is an isosceles triangle.

Question 5:

ABC and DBC are two isosceles triangles on the same base BC (see Figure). Show that $\angle ABD = \angle ACD$.



Answer 5:

In ΔABC,

$AB = AC$ [∵ Given]

$\angle ABC = \angle ACB$ ... (1) [∵ Angles opposite to equal sides are equal]

In ΔDBC,

$DB = DC$ [∵ Given]

$\angle DBC = \angle DCB$ ... (2) [∵ Angles opposite to equal sides are equal]

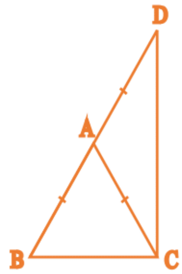
Adding equation (1) and (2), we get

$\angle ABC + \angle DBC = \angle ACB + \angle DCB$

$\Rightarrow \angle ABD = \angle ACD$

Question 6:

ΔABC is an isosceles triangle in which $AB = AC$. Side BA is produced to D such that $AD = AB$ (see Figure). Show that $\angle BCD$ is a right angle.



Answer 6:

In ΔACD,

$AB = AC$ [∵ Given]

$\angle ACD = \angle D$ ... (1) [∵ Angles opposite to equal sides are equal]

In ΔABC,

$AB = AC$ [∵ Given]

$\angle B = \angle ACB$ ... (2) [∵ Angles opposite to equal sides are equal]

In ΔDBC,

$\angle D + \angle B + \angle BCD = 180°$

$\Rightarrow \angle ACD + \angle ACB + \angle BCD = 180°$ [∵ From the equation (1) and (2)]

$\Rightarrow \angle BCD + \angle BCD = 180°$ [∵ $\angle ACD + \angle ACB = \angle BCD$]

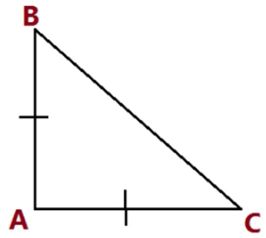
$\Rightarrow 2\angle BCD = 180°$

$\Rightarrow \angle BCD = \frac{180°}{2} = 90°$

Hence, $\angle BCD$ is a right angle.

Question 7:

ABC is a right angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.



Answer 7:

In ΔABC,

AB = AC [∵ Given]

∠B = ∠C [∵ Angles opposite to equal sides are equal]

In ΔABC,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 90° + \angle B + \angle C = 180°$ [∵ ∠A = 90°]

$\Rightarrow 90° + \angle B + \angle B = 180°$ [∵∠C = ∠B]

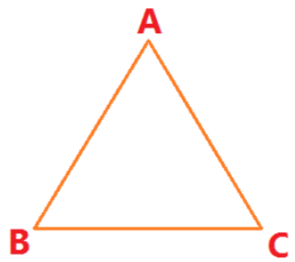
$\Rightarrow 2\angle B = 180° - 90° = 90°$

$\Rightarrow \angle B = \frac{90°}{2} = 45°$

Hence, ∠B = ∠C = 90°

Question 8:

Show that the angles of an equilateral triangle are 60° each.



Answer 8:

In ΔABC,

AB = AC [∵ Given]

∠C = ∠B ... (1) [∵ Angles opposite to equal sides are equal]

Similarly,

In ΔABC,

AB = BC [∵ Given]

∠C = ∠A ... (2) [∵ Angles opposite to equal sides are equal]

From the equation (1) and (2), we have

∠A = ∠B = ∠C ... (3)

In ΔABC,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + \angle A + \angle A = 180°$ [∵ From the equaion (3)]

$\Rightarrow 3\angle A = 180°$

$\Rightarrow \angle A = \frac{180°}{3} = 60°$

Hence, ∠A = ∠B = ∠C = 60°