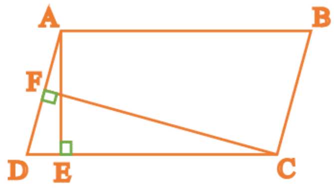
Exercise 9.2

Question 1:

In Figure, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.

Answer 1:



We know that, area of parallelogram = $\frac{1}{2}$ × Base × Altitude

If DC is considered as base, area of ABCD = $\frac{1}{2}$ × DC × AE ... (1)

If AD is considered as base, area of ABCD = $\frac{1}{2}$ × AD × FC ... (2)

From the equation (1) and (2), we have

$\frac{1}{2} \times DC \times AE = \frac{1}{2} \times AD \times FC$

$\Rightarrow DC \times AE = AD \times FC$

$\Rightarrow AB \times AE = AD \times FC$ [∵ DC = AB]

$\Rightarrow 16 \times 8 = AD \times 10$

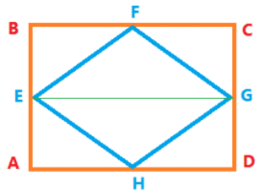
$\Rightarrow AD = \frac{16 \times 8}{10} = 12.8$

Hence, AD = 12.8 cm

Question 2:

If E,F,G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that $ar(\text{EFGH}) = \frac{1}{2}ar(\text{ABCD})$.

Answer 2:



AB = CD [∵ Given]

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow BE = CG$ ... (1) [∵ E and G are the mid-points of sides AB and CD respectively]

And BE || CG ... (2) [∵ AB || CD]

From the equation (1) and (2), BEGC is a parallelogram.

Hence, $ar(\text{GEF}) = \frac{1}{2}ar(\text{BEGC})$ ... (3)

[∵ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

Similarly, AB = CD [∵ Given]

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow AE = DG$ ... (4) [∵E and G are the mid-points of sides AB and CD respectively]

And AE || DG ... (5) [∵ AB || CD]

From the equation (4) and (5), BEGC is a parallelogram.

Hence, $ar(\text{GEH}) = \frac{1}{2}ar(\text{ADGE})$... (6)

Adding equation (3) and (6), we get

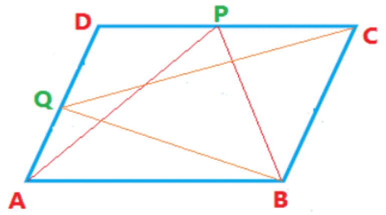
$ar(\text{GEF}) + ar(\text{GEH}) = \frac{1}{2}ar(\text{BEGC}) + \frac{1}{2}ar(\text{ADGE})$

$\Rightarrow ar(\text{EFGH}) = \frac{1}{2}ar(\text{ABCD})$

Question 3:

P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that $ar(\text{APB}) = ar(\text{BQC})$.

Answer 3:



Triangle ABP and parallelogram ABCD lie on the same base AB and between same parallels, AB || CD.

Hence, $ar(\text{APB}) = \frac{1}{2}ar(\text{ABCD})$ ... (1)

[∵ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

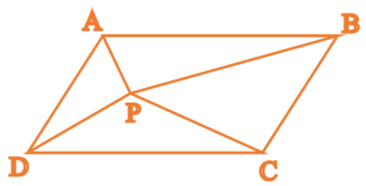
Similarly,

Triangle BQC and parallelogram ABCD lie on the same base AB and between same parallels, AD || BC.

Hence, $ar(\text{BQC}) = \frac{1}{2}ar(\text{ABCD})$ ... (2)

From the equation (1) and (2), $ar(\text{APB}) = ar(\text{BQC})$

Question 4:



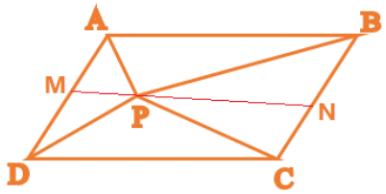
In Figure, P is a point in the interior of a parallelogram ABCD. Show that

(i) $ar(\text{APB}) + ar(\text{PCD}) = \frac{1}{2}ar(\text{ABCD})$

(ii) $ar(\text{APD}) + ar(\text{PBC}) = ar(\text{APB}) + ar(\text{PCD})$

[*Hint*: Through P, draw a line parallel to AB.]

Answer 4:



(i) Construction: Passing through P and parallel to AB, draw a line MPN.

AB || MN ... (1) [∵ By construction]

And AM || BN ... (2) [∵ AD || BC]

From the equation (1) and (2), we get

ABNM is a parallelogram.

Hence, $ar(\text{APB}) = \frac{1}{2}ar(\text{ABNM})$ ... (3)

[∵ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

Similarly,

DC || MN ... (4) [∵ By construction]

And DM || CN ... (5) [∵ AD || BC]

From the equation (4) and (5), we get

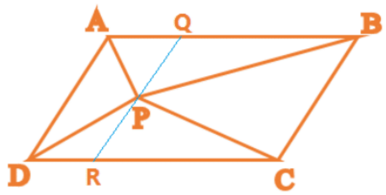
MDCN is a parallelogram.

Hence, $ar(\text{PCD}) = \frac{1}{2}ar(\text{MDCN})$ ... (6)

Adding equation (3) and (6), we get

$$ar(\text{APB}) + ar(\text{PCD}) = \frac{1}{2}ar(\text{ABNM}) + \frac{1}{2}ar(\text{MDCN})$$

$\Rightarrow ar(\text{APB}) + ar(\text{PCD}) = \frac{1}{2}ar(\text{ABCD})$ ... (7)



(ii) Construction: Passing through P and parallel to AD, draw a line QPR.

AD || QR ... (8) [∵ By construction]

And AQ || DR ... (9) [∵ AB || DC]

From the equation (8) and (9), we get

AQRD is a parallelogram.

Hence, $ar(\text{APD}) = \frac{1}{2} ar(\text{AQRD})$ ... (10)

[∵ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

Similarly,

BC || QR ... (11) [∵ By construction]

And QB || RC ... (12) [∵ AB || DC]

From the equation (11) and (12), we get

BCRQ is a parallelogram.

Hence, $ar(\text{PBC}) = \frac{1}{2} ar(\text{BCRQ})$ ... (13)

Adding equation (10) and (13), we get

$$ar(\text{APD}) + ar(\text{PBC}) = \frac{1}{2} ar(\text{AQRD}) + \frac{1}{2} ar(\text{BCRQ})$$

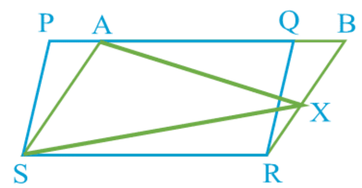
$\Rightarrow ar(\text{APD}) + ar(\text{PBC}) = \frac{1}{2} ar(\text{ABCD})$ ... (14)

From the equation (7) and (14), we get

$ar(\text{APD}) + ar(\text{PBC}) = ar(\text{APB}) + ar(\text{PCD})$

Question 5:

In Figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that



(i) $ar(\text{PQRS}) = ar(\text{ABRS})$

(ii) $ar(\text{AXS}) = \frac{1}{2} ar(\text{PQRS})$

Answer 5:

(i) Parallelogram PQRS and ABRS are lying on the same base RS and between same parallels, SR || PB.

Hence, $ar(\text{PQRS}) = ar(\text{ABRS})$ ... (1)

[∵Parallelograms on the same base (or equal bases) and between the same parallels are equal in area.]

(ii) Triangle AXS and parallelogram ABRS are on the same base AS and between same parallels AS || BR.

Hence, $ar(\text{AXS}) = \frac{1}{2} ar(\text{ABRS})$ ... (2)

[∵ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

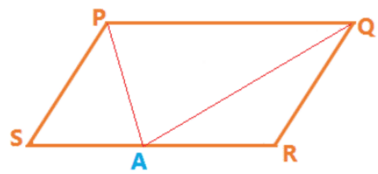
From the equation (1) and (2), we get

$$ar(\text{AXS}) = \frac{1}{2} ar(\text{PQRS})$$

Question 6:

A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Answer 6:



The field is divided into 3 parts: ΔAPS, ΔAPQ and ΔARQ

Triangle APQ and parallelogram PQRS are lying on the same base PQ and between same parallels, PQ || SR.

Hence, $ar(\text{APQ}) = \frac{1}{2} ar(\text{PQRS})$

[∵ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

Hence, the farmer will sow one crop in ΔAPQ and the other crop in the remaining two triangles ΔASP and ΔARQ.