Mathematics

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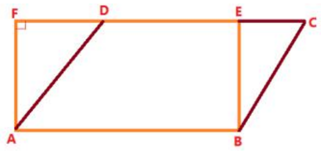
(Chapter – 9)(Areas of Parallelograms and Triangles)

(Class – 9)

Exercise 9.4 (Optional)

Question 1:

Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.



Answer 1:

In ΔAFD,

∠F = 90° [∵ Angle of a rectangle]

AD > AF [∵ In a right triangle, hypotenuse is the longest side]

Adding AB on both the sides, AD + AB > AF + AB

Multiplying both sides by 2,

2[AD + AB] > 2[AF + AB]

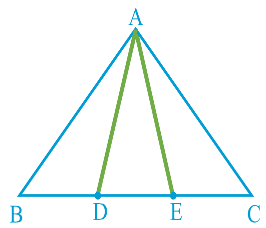
⇒ Perimeter of parallelogram > Perimeter of rectangle

Question 2:

In Figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC).

Can you now answer the question that you have left in the 'Introduction' of this chapter, whether the field of *Budhia* has been actually divided into three parts of equal area?

[**Remark:** Note that by taking BD = DE = EC, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into n equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide ΔABC into n triangles of equal areas.]



Answer 2:

In ΔABE, AD is median. [∵ BD = DE]

Hence, $ar(\text{ABD}) = ar(\text{AED})$ ... (1)

[∵ A median of a triangle divides it into two triangles of equal areas.]

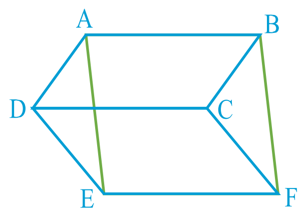
Similarly, in ΔADC, AE is median. [∵ DE = EC]

Hence, $ar(\text{ADE}) = ar(\text{AEC})$ ... (2)

From the equation (1) and (2), $ar(\text{ABD}) = ar(\text{ADE}) = ar(\text{AEC})$

Question 3:

In Figure, ABCD, DCFE and ABFE are parallelograms. Show that ar (ADE) = ar (BCF).



Answer 3:

In ΔADE and ΔBCF,

AD = BC [∵ Opposite sides of parallelogram ABCD]

DE = CF [∵ Opposite sides of parallelogram DCFE]

AE = BF [∵ Opposite sides of parallelogram ABFE]

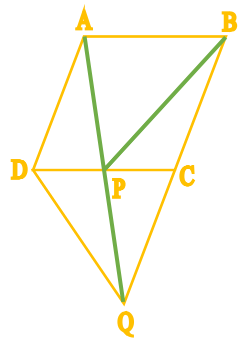
Hence, ΔADE ≅ ΔBCF [∵ SSS Congruency rule]

Hence, $ar(\text{ADE}) = ar(\text{BCF})$ [∵ Congruent triangles are equal in area also]

Question 4:

In Figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that ar (BPC) = ar (DPQ).

[*Hint* : Join AC.]



Answer 4:

In ΔADP and ΔQCP,

∠APD = ∠QPC [∵ Vertically Opposite Angles]

∠ADP = ∠QCP [∵ Alternate angles]

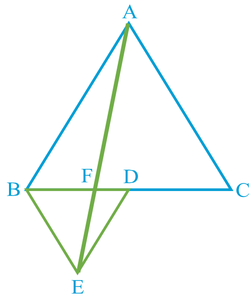
AD = CQ [∵ Given]

Hence, $\Delta ABD \cong \Delta ACD$ [∵ AAS Congruency rule]
Therefore, DP = CP [∵ CPCT]
In ΔCDQ, QP is median. [∵ DP = CP]
Hence, $ar(DPQ) = ar(QPC)$ ... (1)
[∵ A median of a triangle divides it into two triangles of equal areas.]

Similarly,
In ΔPBQ, PC is median. [∵ AD = CQ and AD = BC ⇒ BC = QC]
Hence, $ar(QPC) = ar(BPC)$ ... (2)
From the equation (1) and (2),
$ar(BPC) = ar(DPQ)$

Question 5:

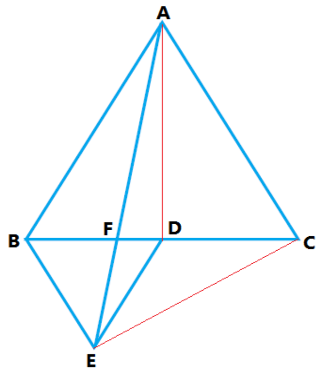
In Figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that



(i) $ar(BDE) = \frac{1}{4} ar(ABC)$ **(ii)** $ar(BDE) = \frac{1}{2} ar(BAE)$
(iii) $ar(ABC) = 2\,ar(BEC)$ **(iv)** $ar(BFE) = ar(AFD)$
(v) $ar(BFE) = 2\,ar(FED)$ **(vi)** $ar(FED) = \frac{1}{8} ar(AFC)$
[*Hint*: Join EC and AD. Show that BE || AC and DE || AB, etc.]

Answer 5:

(i) Construction: Join EC and AD.
Let, $BC = x$
Therefore, $ar(ABC) = \frac{\sqrt{3}}{4} x^2$ [∵ Area of equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$]
And $ar(BDE) = \frac{\sqrt{3}}{4}\left(\frac{x}{2}\right)^2$ [∵ D is mid-point of BC]
$= \frac{1}{4}\left[\frac{\sqrt{3}}{4} x^2\right] = \frac{1}{4}[ar(ABC)]$



(ii) In ΔBEC, ED is median. [∵ D is mid-point of BC]
Hence, $ar(BDE) = \frac{1}{2} ar(BEC)$ ... (1)
[∵ A median of a triangle divides it into two triangles of equal areas.]
∠EBC = 60° and ∠BCA = 60° [∵ Angles of equilateral triangles]
Therefore, ∠EBC =∠BCA
Here, Alternate angles (∠EBC =∠BCA) are equal, Hence, BE || AC
Triangles BEC and BAE are on the same base BE and between same parallels, BE || AC.
Hence, $ar(BEC) = ar(BAE)$ ... (2)
[∵ Triangles on the same base (or equal bases) and between the same parallels are equal in]
From the equation (1) and (2),
$ar(BDE) = \frac{1}{2} ar(BAE)$

(iii) In ΔBEC, ED is median. [∵ D is mid-point of BC]
Hence, $ar(BDE) = \frac{1}{2} ar(BEC)$ ... (3)
[∵ A median of a triangle divides it into two triangles of equal areas.]
$ar(BDE) = \frac{1}{4} ar(ABC)$ ... (4) [∵ Proved above in **(i)**]
From the equation (3) and (4),
$ar(ABC) = 2\,ar(BEC)$

(iv) ∠ABD = 60° and ∠BDE = 60°   [∵ Angles of equilateral triangle]
Therefore, ∠ABD =∠BDE
Here, Alternate angles (∠ABD =∠BDE) are equal,
Hence, BA || ED
Triangles BDE and AED are on the same base ED and between same parallels BA || ED.
Hence, $ar(\text{BDE}) = ar(\text{AED})$
[∵ Triangles on the same base (or equal bases) and between the same parallels are equal in]
Subtracting $ar(\text{FED})$ form both the sides
$ar(\text{BDE}) - ar(\text{FED}) = ar(\text{AED}) - ar(\text{FED})$
$\Rightarrow ar(\text{BEF}) = ar(\text{AFD})$

(v) In ΔBEC, $AD^2 = AB^2 - BD^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4} \Rightarrow AD = \frac{\sqrt{3}a}{2}$

In ΔLED, $EL^2 = DE^2 - DL^2 = \left(\frac{a}{2}\right)^2 - \left(\frac{a}{4}\right)^2 = \frac{a^2}{4} - \frac{a^2}{16} = \frac{3a^2}{16} \Rightarrow EL = \frac{\sqrt{3}a}{4}$

Therefore, $ar(\text{AFD}) = \frac{1}{2} \times \text{FD} \times \text{AD} = \frac{1}{2} \times \text{FD} \times \frac{\sqrt{3}a}{2}$ ... (5)

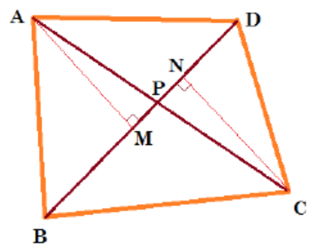
And $ar(\text{EFD}) = \frac{1}{2} \times \text{FD} \times \text{EL} = \frac{1}{2} \times \text{FD} \times \frac{\sqrt{3}a}{4}$ ... (6)

From the equation (5) and (6),
$ar(\text{AFD}) = 2\, ar(\text{FED})$ ... (7)
$\Rightarrow ar(\text{BFE}) = 2\, ar(\text{FED})$ [∵ Comparing with **(iv)**]

(vi) $ar(\text{BDE}) = \frac{1}{4}\, ar(\text{ABC})$ [∵ From the equation **(i)**]
$\Rightarrow ar(\text{BEF}) + ar(\text{FED}) = \frac{1}{4}\, ar(\text{ABC})$
$\Rightarrow ar(\text{BEF}) + ar(\text{FED}) = \frac{1}{4}[2\, ar(\text{ADC})]$ [∵ $ar(\text{ABC}) = 2\, ar(\text{ABC})$]
$\Rightarrow 2\, ar(\text{FED}) + ar(\text{FED}) = \frac{1}{2}[ar(\text{ADC})]$ [∵ From the equation **(v)**]
$\Rightarrow 3\, ar(\text{FED}) = \frac{1}{2}[ar(\text{AFC}) - ar(\text{AFD})]$
$\Rightarrow 3\, ar(\text{FED}) = \frac{1}{2}[ar(\text{AFC}) - 2ar(\text{FED})]$ [∵ From the equation (7)]
$\Rightarrow 3\, ar(\text{FED}) = \frac{1}{2} ar(\text{AFC}) - \frac{1}{2} \times 2ar(\text{FED})$
$\Rightarrow 3\, ar(\text{FED}) = \frac{1}{2} ar(\text{AFC}) - ar(\text{FED})$
$\Rightarrow 4\, ar(\text{FED}) = \frac{1}{2} ar(\text{AFC})$
$\Rightarrow ar(\text{FED}) = \frac{1}{8} ar(\text{AFC})$

Question 6:
Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that ar (APB) × ar (CPD) = ar (APD) × ar (BPC). [*Hint*: From A and C, draw perpendiculars to BD.]

Answer 6:
Construction: From the points A and C, draw perpendiculars AM and CN on BD.



$ar(\text{APB}) \times ar(\text{CPD}) = \frac{1}{2} \times \text{BP} \times \text{AM} \times \frac{1}{2} \times \text{PD} \times \text{CN}$ ... (1)

$ar(\text{APD}) \times ar(\text{BPC}) = \frac{1}{2} \times \text{PD} \times \text{AM} \times \frac{1}{2} \times \text{BP} \times \text{CN}$ ... (2)

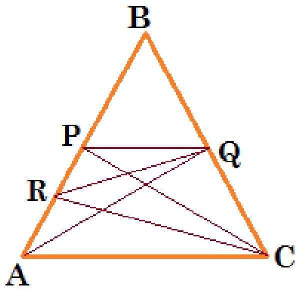
From the equation (1) and (2), $ar(\text{APB}) \times ar(\text{CPD}) = ar(\text{APD}) \times ar(\text{BPC})$

Question 7:

P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that

(i) $ar(\text{PRQ}) = \frac{1}{2} ar(\text{ARC})$ **(ii)** $ar(\text{RQC}) = \frac{3}{8} ar(\text{ABC})$ **(iii)** $ar(\text{PBQ}) = \ ar(\text{ARC})$

Answer 7:



Construction: Join AQ, PC, RC and RQ.

(i) In ΔAPQ, QR is median. [∵ Given]

Hence, $ar(\text{PQR}) = \frac{1}{2} ar(\text{APQ})$ ... (1)

[∵ A median of a triangle divides it into two triangles of equal areas.]

Similarly,

In ΔAQB, QP is median. [∵ Given]

Hence, $ar(\text{APQ}) = \frac{1}{2} ar(\text{ABQ})$ ... (2)

And, in ΔABC, AQ is median. [∵ Given]

Hence, $ar(\text{ABQ}) = \frac{1}{2} ar(\text{ABC})$ ... (3)

From the equation (1), (2) and (3),

$ar(\text{PQR}) = \frac{1}{8} ar(\text{ABC})$ ... (4)

In ΔARC, CR is median. [∵ Given]

Hence, $ar(\text{ARC}) = \frac{1}{2} ar(\text{APC})$ ... (5)

[∵ A median of a triangle divides it into two triangles of equal areas.]

Similarly,

In ΔABC, CP is median. [∵ Given]

Hence, $ar(\text{APC}) = \frac{1}{2} ar(\text{ABC})$ ... (6)

From the equation (5) and (6),

$ar(\text{ARC}) = \frac{1}{4} ar(\text{ABC})$ ... (7)

From the equation (4) and (7),

$$ar(\text{PQR}) = \frac{1}{8} ar(\text{ABC}) = \frac{1}{2}\left[\frac{1}{4} ar(\text{ABC})\right] = \frac{1}{2} ar(\text{ARC})$$

(ii) $ar(\text{RQC}) = ar(\text{RQA}) + ar(\text{AQC}) - ar(\text{ARC})$ ... (8)

In ΔPQA, QR is median. [∵ Given]

Hence, $ar(\text{RQA}) = \frac{1}{2} ar(\text{PQA})$ ... (9)

In ΔAQB, PQ is median.

Hence, $ar(\text{PQA}) = \frac{1}{2} ar(\text{AQB})$ ... (10)

In ΔABC, AQ is median. [∵ Given]

Hence, $ar(\text{AQB}) = \frac{1}{2} ar(\text{ABC})$ ... (11)

From the equation (9), (10) and (11),

$ar(\text{RQA}) = \frac{1}{8} ar(\text{ABC})$ ... (12)

In ΔABC, AQ is median. [∵ Given]

Hence, $ar(\text{AQC}) = \frac{1}{2} ar(\text{ABC})$ ... (13)

In ΔAPC, CR is median.

Hence, $ar(\text{ARC}) = \frac{1}{2} ar(\text{APC})$ ... (14)

In ΔABC, CP is median. [∵ Given]

Hence, $ar(\text{APC}) = \frac{1}{2}ar(\text{ABC})$ ... (15)

From the equation (14) and (15),

$ar(\text{ARC}) = \frac{1}{4}ar(\text{ABC})$ ... (16)

From the equation (8), (12), (13) and (16),

$$ar(\text{RQC}) = \frac{1}{8}ar(\text{ABC}) + \frac{1}{2}ar(\text{ABC}) - \frac{1}{4}ar(\text{ABC}) = \frac{3}{8}ar(\text{ABC})$$

(iii) In ΔABQ, PQ is median. [∵ Given]

Hence, $ar(\text{PBQ}) = \frac{1}{2}ar(\text{ABQ})$ ... (17)

In ΔABC, AQ is median.

Hence, $ar(\text{ABQ}) = \frac{1}{2}ar(\text{ABC})$ ... (18)

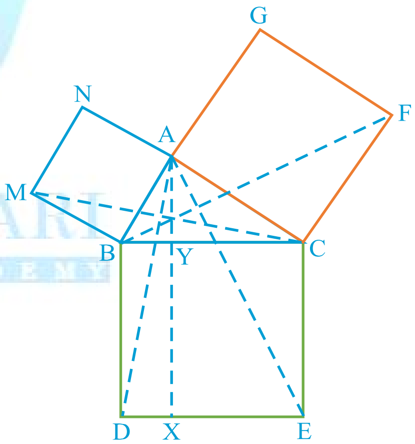
From the equation (16), (17) and (18),

$ar(\text{PQB}) = ar(\text{ARC})$

Question 8:

In Figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX⊥ DE meets BC at Y. Show that:

(i) ΔMBC ≅ ΔABD
(ii) $ar(\text{BYXD}) = 2\ ar(\text{MBC})$
(iii) $ar(\text{BYXD}) = ar(\text{ABMN})$
(iv) ΔFCB ≅ ΔACE
(v) $ar(\text{CYXE}) = 2\ ar(\text{FCB})$
(vi) $ar(\text{CYXE}) = ar(\text{ACFG})$
(vii) $ar(\text{BCED}) = ar(\text{ABMN}) + ar(\text{ACFG})$



Answer 8:

(i) In ΔMBC and ΔABD,

AB = AC [∵ Sides of square]

∠MBC = ∠ABD [∵ Each 90° + ∠ABC]

MB = AB [∵ Sides of square]

Hence, ΔMBC ≅ ΔABD [∵ SAS Congruency rule]

(ii) Triangle ABD and parallelogram BYXD are on the same base BD and between same parallels AX || BD.

Hence, $ar(\text{ABD}) = \frac{1}{2}ar(\text{BYXD})$ ... (1)

[∵ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

But, ΔMBC ≅ ΔABD [∵ Proved above]

Therefore, $ar(\text{MBC}) = ar(\text{ABD})$ ... (2)

From the equation (1) and (2),

$ar(\text{MBC}) = \frac{1}{2}ar(\text{BYXD})$ ... (3)

$\Rightarrow 2\ ar(\text{MBC}) = ar(\text{BYXD})$

(iii) Triangle MBC and square ABMN are on the same base MB and between same parallels MB || NC.

Hence, $ar(\text{MBC}) = \frac{1}{2} ar(\text{ABMN})$ ... (4)

[∵ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

From the equation (3) and (4),

$ar(\text{BYXD}) = ar(\text{ABMN})$

(iv) In ΔACE and ΔBCF,

CE = BC [∵ Sides of square]

∠ACE = ∠BCF [∵ Each 90° + ∠BCA]

AC = CF [∵ Sides of square]

Hence, ΔACE ≅ ΔBCF [∵ SAS Congruency rule]

(v) Triangle ACE and square CYXE are on the same base CE and between same parallels CE || AX.

Hence, $ar(\text{ACE}) = \frac{1}{2} ar(\text{CYXE})$

[∵ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

$\Rightarrow ar(\text{FCB}) = \frac{1}{2} ar(\text{CYXE})$ ... (5) [∵ $ar(\text{FCB}) = ar(\text{ACE})$]

$\Rightarrow 2\, ar(\text{FCB}) = ar(\text{CYXE})$

(vi) Triangle BCF and square ACFG are on the same base CF and between same parallels CF || FG.

Hence, $ar(\text{BCF}) = \frac{1}{2} ar(\text{ACFG})$ ... (6)

[∵ If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

From the equation (5) and (6),

$\Rightarrow ar(\text{CYXE}) = ar(\text{ACFG})$

(vii) From the result of **(iii)**, we have

$ar(\text{BYXD}) = ar(\text{ABMN})$ ... (7)

From the result of **(vi)**, we have

$ar(\text{CYXE}) = ar(\text{ACFG})$ ... (8)

Adding (7) and (8), we get

$ar(\text{BYXD}) + ar(\text{CYXE}) = ar(\text{ABMN}) + ar(\text{ACFG})$

$\Rightarrow ar(\text{BCED}) = ar(\text{ABMN}) + ar(\text{ACFG})$